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# Vector potential quantization and the photon wave-particle representation

C Meis<sup>1\*</sup> and P R Dahoo<sup>2</sup>

<sup>1</sup> CEA Saclay. National Institute for Nuclear Science and Technology,  
Université Paris Saclay – 91191 Gif-sur-Yvette, France.

<sup>2</sup> LATMOS /IPSL, UVSQ Université Paris-Saclay, UPMC Univ. Paris 06, CNRS,  
F-78280, Guyancourt, France.

E-mail: [constantin.meis@cea.fr](mailto:constantin.meis@cea.fr)

**Abstract.** The quantization procedure of the vector potential is enhanced at a single photon state revealing the possibility for a simultaneous representation of the wave-particle nature of the photon. Its relationship to the quantum vacuum results naturally. A vector potential amplitude operator is defined showing the parallelism with the Hamiltonian of a massless particle. It is further shown that the quantized vector potential satisfies both the wave propagation equation and a linear time-dependent Schrödinger-like equation.

## Introduction

We analyse first the fundamental link between the electromagnetic wave theory and quantum electrodynamics (QED) [1-3]. In the classical description issued from Maxwell's equations the energy density of an electromagnetic wave with electric and magnetic fields  $\vec{E}(\vec{r}, t)$  and  $\vec{B}(\vec{r}, t)$  respectively is

$$W_C(\vec{r}, t) = \frac{1}{2} \left( \varepsilon_0 |\vec{E}(\vec{r}, t)|^2 + \frac{1}{\mu_0} |\vec{B}(\vec{r}, t)|^2 \right) \quad (1)$$

where  $\varepsilon_0$  and  $\mu_0$  are the electric permittivity and magnetic permeability of the vacuum.

In the case of a monochromatic plane wave the electric and magnetic fields are proportional to the vector potential amplitude  $A_0(\omega)$  and the energy density writes

$$W_C(\vec{r}, t) = 4\varepsilon_0 \omega^2 |A_0(\omega)|^2 \sin^2(\vec{k} \cdot \vec{r} - \omega t) \quad (2)$$

whose mean value over a period, that is over a wavelength, becomes time and space independent

$$W_C = 2\varepsilon_0 \omega^2 |A_0(\omega)|^2 \quad (3)$$

In the quantum theory the energy density for  $N$  photons with angular frequency  $\omega$  in a volume  $V$  is

$$W_Q = \frac{N\hbar\omega}{V} \quad (4)$$

where  $\hbar = h/2\pi$  is Planck's reduced constant.

In order to link the classical and quantum description it is generally imposed for  $N = 1$ , thus for a single photon state, the relations (3) and (4) to be equal. In this way, the vector potential amplitude is

$$|A_0(\omega)| = \sqrt{\frac{\hbar}{2\varepsilon_0\omega V}} \quad (5)$$

It is worth noting that as a result of this procedure an external arbitrary parameter  $V$  has been introduced in the last equation which is supposed to express naturally the photon vector potential amplitude, an intrinsic physical property. Nevertheless, this equation is used to define the fundamental link relations between the classical and quantum theory of light through the definition of the vector potential amplitude operators for a photon

$$A_{k\lambda} = \sqrt{\frac{\hbar}{2\varepsilon_0\omega_k V}} a_{k\lambda} \quad A_{k\lambda}^* = \sqrt{\frac{\hbar}{2\varepsilon_0\omega_k V}} a_{k\lambda}^+ \quad (6)$$

where  $a_{k\lambda}$  and  $a_{k\lambda}^+$  are the annihilation and creation operators respectively for a  $k$ -mode and  $\lambda$ -polarization photon with angular frequency  $\omega_k$ .

### Vector potential in QED

It is useful to examine how the last relations are used in QED calculations [1-4]. The vector potential operator writes generally as a superposition of the vector potentials of all the  $k$ -modes and  $\lambda$  polarization photons with polarization vector  $\hat{\varepsilon}_\lambda$

$$\vec{A}(\vec{r}, t) = \sum_{k,\lambda} \sqrt{\frac{\hbar}{2\varepsilon_0\omega_k V}} \left[ a_{k\lambda} \hat{\varepsilon}_\lambda e^{i(\vec{k}\cdot\vec{r} - \omega_k t + \theta)} + a_{k\lambda}^+ \hat{\varepsilon}_\lambda^* e^{-i(\vec{k}\cdot\vec{r} - \omega_k t + \theta)} \right] \quad (7)$$

The discrete summation over the modes  $k$  is generally replaced by a continuous one over the angular frequencies  $\omega$  following the transformation issued from the density of states theory

$$\sum_{k,\lambda} \rightarrow \frac{V}{2\pi^2 c^3} \sum_{\lambda} \int \omega^2 d\omega \quad (8)$$

where  $c$  is the velocity of light in vacuum and  $\lambda$  takes two values corresponding to the Left and Right hand circular polarizations. Consequently, for all the calculations involving the square of the amplitude of the vector potential this mathematical operation helps to eliminate the volume parameter  $V$ . However, the relation (8) has been obtained under the condition that all the photons wavelengths are much smaller than the characteristic dimensions of the volume  $V$ .

### Single photon state vector potential and the quantum vacuum

The methodology presented above gives quite physical results when considering a system of photons within a volume with dimensions much bigger compared to their wavelengths. However, the experimental evidence [1-4] has shown that a single photon is an indivisible entity with definite energy and momentum. Despite of this the relation (6) gives no precise information on its vector potential amplitude.

Now, the energy density of the electromagnetic waves in the classical description as well as in QED depends on the fourth power of the angular frequency  $\omega^4$  [1,3]. Consequently, the relation (3) entails automatically that the vector potential amplitude is proportional to  $\omega$ . Indeed, the unit analysis of the general solution of Maxwell's equations for the vector potential shows that it is inversely proportional to time, thus proportional to an angular frequency. Consequently, for a  $k$ - mode photon the vector potential amplitude  $\alpha_0$  can be written as [4,5]

$$\alpha_{0_k} = \xi \omega_k \quad (9)$$

where  $\xi$  is a constant.

Thus, the fundamental physical quantities characterizing both the wave and particle nature of a single photon: energy and momentum (particle), vector potential amplitude and wave vector (wave), are all related to the angular frequency as follows [4,5]

$$\frac{E_k}{\hbar} = \frac{|\vec{p}_k|}{\hbar/c} = \frac{\alpha_{0_k}}{\xi} = |\vec{k}|c = \omega_k \quad (10)$$

In the plane wave representation the vector potential for a  $k$ -mode and  $\lambda$  polarization photon can be expressed over a period and repeated successively along the propagation axis as

$$\vec{\alpha}_{k,\lambda}(\vec{r}, t) = \omega_k \left[ \xi \hat{\epsilon}_\lambda e^{i(\vec{k} \cdot \vec{r} - \omega_k t + \theta)} + cc \right] = \omega_k \vec{\Xi}_{k\lambda}(\omega_k, \vec{r}, t) \quad (11)$$

with  $cc$  the complex conjugate and has to satisfy the wave propagation equation

$$\vec{\nabla}^2 \vec{\alpha}_{k,\lambda}(\vec{r}, t) - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{\alpha}_{k,\lambda}(\vec{r}, t) = 0 \quad (12)$$

leading to

$$\left[ \alpha_0^2 + \xi^2 c^2 \vec{\nabla}^2 \right] \vec{\alpha}_{k,\lambda}(\vec{r}, t) = 0 \quad (13)$$

The last expression entails that the vector potential amplitude of the photon can be expressed as an operator

$$\tilde{\alpha}_0 = -i\xi c \vec{\nabla} \quad (14)$$

which is quite symmetrical to the relativistic Hamiltonian operator for a massless particle

$$\tilde{H} = -i\hbar c \vec{\nabla} \quad (15)$$

Applying the vector potential amplitude operator (14) upon the vector potential expression (11) we get finally

$$i\xi \frac{\partial}{\partial t} \vec{\alpha}_{k,\lambda}(\vec{r}, t) = \tilde{\alpha}_0 \vec{\alpha}_{k,\lambda}(\vec{r}, t) \quad (16)$$

This is a quantum equation for the vector potential with quantization constant  $\xi$  analogue to Schrödinger's equation for the energy with quantization constant  $\hbar$ . Hence, we get the coupled equation for a photon in a non-local representation with the vector potential as a photon wave function

$$i \left( \frac{\xi}{\hbar} \right) \frac{\partial}{\partial t} \vec{\alpha}_{k,\lambda}(\vec{r}, t) = \left( \frac{\tilde{\alpha}_0}{\tilde{H}} \right) \vec{\alpha}_{k,\lambda}(\vec{r}, t) \quad (17)$$

In a first approximation the value of the constant  $\xi$  has been evaluated to be [6,7]

$$|\xi| \propto \left| \frac{1}{(2\pi)^{3/2}} \sqrt{\frac{\hbar}{8\alpha_{FS}\epsilon_0 c^3}} \right| = \left| \frac{\hbar}{4\pi e c} \right| = 1.747 \cdot 10^{-25} \text{ Volt } m^{-1} s^2 \quad (18)$$

where  $\alpha_{FS}=1/137$  is the Fine Structure constant and  $e$  is the electron charge.

Now, at very low frequencies the wavelength tends to infinity and the vector potential to zero but the function  $\vec{\Xi}_{k\lambda}(\omega_k, \vec{r}, t)$  composing the vector potential does not vanish and tends to a unique expression for all modes

$$\vec{\Xi}^{(0)} = \xi \hat{\mathcal{E}}_\lambda e^{i\theta} + cc \quad (19)$$

Consequently, in absence of photons  $\vec{\Xi}^{(0)}$  is a real field with amplitude  $\xi$  having electric units and permeating all space. Thus, it can be characterized as a component of the quantum vacuum. This means that the electromagnetic waves, that is photons, are oscillations of the vacuum field  $\vec{\Xi}^{(0)}$ .

### Conclusion and discussion

We have seen here that the quantization of the vector potential amplitude enhanced at a single photon state (9) complements the fundamental relations of the photon (10) and leads to the coupled equation (17) for which the vector potential (11) with the quantized amplitude  $\xi\omega$  behaves as a real wave function. The quantization constant  $\xi$  of the photon vector potential amplitude has electric essence and derives from the quantum vacuum. Consequently, the vacuum is not a sea of photons, which leads to the well-known QED singularity of infinite vacuum energy [8] (so called *quantum vacuum catastrophe*), but the electromagnetic waves (photons) are waves of the quantum vacuum sea which is composed of a real potential field  $\vec{\Xi}^{(0)}$ . The relation (17) indicates that a vibration of the vacuum field at an angular frequency  $\omega$  gives rise to a photon with vector potential amplitude  $\xi\omega$  and energy  $\hbar\omega$ .

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